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Neutrino masses and leptonic CP violation

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Neutrino oscillations as solutions of the solar neutrino problems and the atmospheric neutrino deficits may restrict neutrino mass squared differences and mixing angles in the three-neutrino mixing scheme. Currently we have several solutions depending on the interpretation of the solar neutrino problem. Combining the neutrino oscillation solutions and a mass matrix ansatz, we investigate the neutrino mass bounds and find a possible leptonic CP-violating rephasing-invariant quantity $J^l_{CP} \leqslant 0.012$ for the large mixing angle MSW and just-so vacuum oscillation solutions, and $J^l_{CP} \leqslant 0.0013$ for the small mixing angle MSW solution.

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I. INTRODUCTION

The quark masses and the related flavor mixings are the most intriguing riddles to be understood in the standard model (SM). Within the SM, the quark masses and flavor mixing angles are not predictable. The flavor mixing angles arise since the quark states for which the weak interaction is diagonal are not mass eigenstates. Moreover, degenerate quarks of a given charge render the flavor mixing angles physically meaningless. Thanks to this fact, we get a hint that the flavor mixing angles can be related to the elements of the quark mass matrix. As an attempt to provide a relationship between the flavor mixing angles and the elements of quark mass matrix, the mass matrix ansatz has been suggested [1,2]. With the help of a mass matrix ansatz, we may predict some free parameters in the SM.

On the other hand, recent neutrino experimental results and cosmological observations provide evidence for nonzero neutrino masses and possible lepton flavor mixings. Then, the SM has to be enlarged and we have more free parameters to describe all fermion masses and their mixing angles. In this case, one may also reduce the number of free parameters by using separate lepton mass matrix ansatz. If it is possible to provide some quark-lepton symmetry in the quark and lepton mass matrices, one may reduce the number of free parameters much more. In recent work [3], we showed that the hierarchical quark mixing pattern as well as bimaximal lepton mixing pattern can arise from one single particular mass matrix based on the permutation symmetry with suitable breaking. Remarkably those different mixing patterns could be generated by using the same texture of the mass matrices for quarks and leptons but with different hierarchies. However, although nonzero neutrino masses and mixings can be interpreted as a solution to the solar [4] and the atmospheric [5] neutrino anomalies, the present neutrino experimental results do not pin down the values of neutrino masses and mixing angles in three-neutrino oscillation scheme. Moreover, the solution for the solar neutrino deficit may be either small or large mixing with different mass squared differences depending on whether or not we consider the matter effect [Mikheyev-Smirnov-Wolfensten (MSW) effect] [6,7]. Thus, one can at best estimate the hierarchy of neutrino mass patterns and their mixings case by case. In this work, we show in more detail how the lepton flavor mixing matrix can be obtained from a specific form of lepton mass matrix by assuming quark-lepton symmetry for the fermion mass matrix. For complete discussion, we will take into account both large and small mixing angle solutions for the solar neutrino deficit combined with the large mixing solution for the Super-Kamiokande atmospheric neutrino anomaly.

In addition, it will be very interesting to study the possible CP violation in the lepton sector, which arises due to the nonvanishing CP phase in the flavor mixing matrix [8]. To do this, we will calculate the invariant leptonic CP violating quantity J^l_{CP} [9] from the phenomenological lepton flavor mixing matrix. As will be shown later, the invariant quantity, J^l_{CP}, depends on the neutrino masses as well as the CP phase. From the estimate of the neutrino mass bounds based on the neutrino experimental results, we will provide the possible range of J^l_{CP}.

II. NEUTRINO MIXING MATRIX WITH A CP VIOLATING PHASE

Let us begin by assuming that the form of lepton mass matrix can be derived from the mass matrix ansatz based on

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the permutation symmetry with suitable breaking which is used in the quark sector [3]. As shown in Ref. [2], the mass matrix has the following form:

$$M_H=\begin{pmatrix} 0 & A & 0 \\ A & D & B \\ 0 & B & C \end{pmatrix}. \tag{1}$$

The parameters A,B,C and D can be expressed in terms of the fermion mass eigenvalues and one free parameter ϵ^l. One can take the mass eigenvalues to be $-m_1$, m_2, and m_3 with the following three conditions:

$$\mathrm{Tr}(M_H)=-m_1+m_2+m_3,$$

$$\mathrm{Det}(M_H)=-m_1m_2m_3,$$

and

$$\mathrm{Tr}(M_H^2)=m_1^2+m_2^2+m_3^2. \tag{2}$$

The sign of the fermion mass is irrelevant since it can be changed by a chiral transformation. From those relations, we obtain the following form of fermion mass matrix:

$$M=\begin{pmatrix} 0 & \sqrt{\dfrac{m_1m_2m_3}{m_3-\epsilon^l}} & 0 \\ \sqrt{\dfrac{m_1m_2m_3}{m_3-\epsilon^l}} & m_2-m_1+\epsilon^l & \zeta(m_2-m_1+\epsilon^l) \\ 0 & \zeta(m_2-m_1+\epsilon^l) & m_3-\epsilon^l \end{pmatrix}, \tag{3}$$

in which the analytic relation between two parameters ϵ^l and ζ is given [3] by

$$\zeta^2=\frac{\epsilon^l(m_3-m_2+m_1-\epsilon^l)(m_3-\epsilon^l)-\epsilon^l m_1m_2}{(m_3-\epsilon^l)(m_2-m_1+\epsilon^l)^2}. \tag{4}$$

With the help of the analytic form of the orthogonal matrix U presented in Ref. [3], the real symmetric mass matrix M can be diagonalized. The mass matrices for charged leptons and neutrinos have the same form of the mass matrix (3). In particular, notice that the parameter ϵ^l will be taken to be identical in both the charged lepton mass matrix and the neutrino mass matrix, and will be determined from the neutrino experimental results. However, the parameters ζ are different in the two mass matrices because they depend on their fermion masses. Then, the neutrino mass matrix M_ν and charged lepton mass matrix M_l can be brought to diagonal forms by the real unitary matrices U_ν and U_l,

$$U_\nu M_\nu U_\nu^\dagger=\mathrm{diag}(-m_1,m_2,m_3),$$

$$U_l M_l U_l^\dagger=\mathrm{diag}(-m_e,m_\mu,m_\tau),$$

where m_1,m_2, and m_3 are neutrino masses from now on. The lepton flavor mixing matrix V_{CKM}^l is related to U_ν and U_l as follows:

$$V_{\mathrm{CKM}}^l=PU_lP^{-1}U_\nu^T, \tag{5}$$

where the phase matrix is $P=\mathrm{diag}(e^{i\delta^l},1,1)$. More generally, we can also take the phase matrix, P, as $\mathrm{diag}(e^{i\delta_1},e^{i\delta_2},e^{i\delta_3})$. One may eliminate the phase δ_3 by a phase transformation of fields. Because of the hierarchy of the charged lepton masses, Eq. (5) contains only the combination of phases in the form, $\delta_1-\delta_2$, which will be identified as δ^l. To see easily how the lepton mixing pattern is related to the lepton mass hierarchy, first of all, we present the lepton flavor mixing matrix *in the leading approximation*. In the next section, the exact form derived from Eq. (5) will be used to determine the magnitudes of the elements of the mixing matrix.

Since the charged lepton family has pronounced mass hierarchy $m_e\ll m_\mu\ll m_\tau$, the charged lepton mass matrix can be presented in the approximate form as

$$M_l\simeq\begin{pmatrix} 0 & \sqrt{m_em_\mu} & 0 \\ \sqrt{m_em_\mu} & m_\mu & \sqrt{\epsilon^l m_\tau} \\ 0 & \sqrt{\epsilon^l m_\tau} & m_\tau \end{pmatrix}. \tag{6}$$

From the unitary transformation, we obtain the approximate form of the matrix U_l as follows:

$$U_l\simeq\begin{pmatrix} 1 & -\sqrt{\dfrac{m_e}{m_\mu}} & 0 \\ \sqrt{\dfrac{m_e}{m_\mu}} & 1 & -\sqrt{\dfrac{\epsilon^l}{m_\tau}} \\ 0 & \sqrt{\dfrac{\epsilon^l}{m_\tau}} & 1 \end{pmatrix}, \tag{7}$$

where we assumed that the parameter $\epsilon^l\ll m_\tau$. On the other hand, the neutrino mass matrix can be obtained from the mixing pattern among three neutrinos and their mass hierarchy. As shown in Ref. [3], the large mixing between ν_μ and ν_τ, which is a solution for the atmospheric neutrino anomaly, can be achieved by taking $\epsilon^l\simeq m_3/2$ and $m_1,m_2\ll m_3$ in Eq. (3). We also note that the large (small) mixing between ν_e and ν_μ, which is a solution for the solar neutrino deficit, can be obtained by taking $m_1\simeq m_2(m_1\ll m_2)$. Keeping the next-to-leading order, the neutrino mass matrix, Eq. (3), becomes

$$M_\nu \simeq \begin{pmatrix} 0 & \sqrt{2m_1m_2} & 0 \\ \sqrt{2m_1m_2} & \frac{m_3}{2}\left(1+2\frac{m_2-m_1}{m_3}\right) & \frac{m_3}{2}\left(1-\frac{m_2-m_1}{m_3}\right) \\ 0 & \frac{m_3}{2}\left(1-\frac{m_2-m_1}{m_3}\right) & \frac{m_3}{2} \end{pmatrix}, \tag{8}$$

and the form of U_ν is approximately given by

$$U_\nu \simeq \begin{pmatrix} w_2\left(1+\frac{m_1}{2m_3}\right) & -\frac{w_1}{\sqrt{2}}\left(1+\frac{m_1}{2m_3}\right) & \frac{w_1}{\sqrt{2}}\left(1-\frac{m_2}{m_3}-\frac{m_1}{2m_3}\right) \\ w_1\left(1-\frac{m_2}{2m_3}\right) & \frac{w_2}{\sqrt{2}}\left(1-\frac{m_2}{2m_3}\right) & -\frac{w_2}{\sqrt{2}}\left(1+\frac{m_2}{2m_3}+\frac{m_1}{m_3}\right) \\ \sqrt{\frac{m_1m_2}{m_3^2}} & \frac{1}{\sqrt{2}}\left(1+\frac{m_2-m_1}{2m_3}\right) & \frac{1}{\sqrt{2}}\left(1-\frac{m_2-m_1}{2m_3}\right) \end{pmatrix}, \tag{9}$$

where

$$w_1 \equiv \sqrt{\frac{m_1}{m_1+m_2}} \text{ and } w_2 \equiv \sqrt{\frac{m_2}{m_1+m_2}} \text{ with } w_1^2+w_2^2=1.$$

From Eqs. (5), (7), and (9), the lepton flavor mixing matrix is expressed in the leading order in terms of the lepton masses, w_1, w_2, and the CP phase δ^l:

$$V^l_{\rm CKM} \simeq \begin{pmatrix} w_2+w_1\sqrt{\frac{m_e}{2m_\mu}}e^{i\delta^l} & w_1-w_2\sqrt{\frac{m_e}{2m_\mu}}e^{i\delta^l} & -\sqrt{\frac{m_e}{2m_\mu}}e^{i\delta^l} \\ -\frac{w_1}{\sqrt{2}}+w_2\sqrt{\frac{m_e}{m_\mu}}e^{-i\delta^l} & \frac{w_2}{\sqrt{2}}+w_1\sqrt{\frac{m_e}{m_\mu}}e^{-i\delta^l} & \frac{1}{\sqrt{2}} \\ \frac{w_1}{\sqrt{2}} & -\frac{w_2}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{10}$$

The CP-violating rephasing-invariant quantity, J^l_{CP}, is presented by

$$J^l_{CP} \equiv Im[V^l_{11}V^{l*}_{12}V^{l*}_{21}V^l_{22}] = w_1w_2\frac{1}{2}\sqrt{\frac{m_e}{2m_\mu}}\sin\delta^l. \tag{11}$$

Now let us express the lepton flavor mixing matrix in the standard parametrization [10]. As is well known, in the quark sector the standard parametrization is given by

$$V^l_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{12}$$

where s_{ij}, c_{ij} stand for $\sin\theta_{ij}$ and $\cos\theta_{ij}$, respectively. One can then relate the elements of the mixing matrix in the standard parametrization to the elements of the flavor mixing matrix (10) by using the fact that the magnitudes of the mixing matrix elements and the Jarlskog rephasing-invariant quantity, J^l_{CP}, are independent of the parametrization. Then, the mixing angles, θ_{ij}, can be expressed by

$$\tan\theta_{12} = \frac{|V^l_{12}|}{|V^l_{11}|} \simeq \sqrt{\frac{w_1^2+w_2^2(m_e/2m_\mu)-2w_1w_2\sqrt{m_e/2m_\mu}\cos\delta^l}{w_2^2+w_1^2(m_e/2m_\mu)+2w_1w_2\sqrt{m_e/2m_\mu}\cos\delta^l}}, \tag{13}$$

$$\sin\theta_{13} = |V^l_{13}| \simeq \sqrt{\frac{m_e}{2m_\mu}}, \tag{14}$$

$$\tan\theta_{23} = \frac{|V^l_{23}|}{|V^l_{33}|} \simeq 1. \tag{15}$$

The magnitude of V^l_{13} can be constrained by the CHOOZ experimental results [11] and it turns out to be small, i.e., $|V^l_{13}|\leqslant 0.22$. Then, the lepton mixing matrix (12) is approximately written as

$$V^l_{\rm CKM}\simeq\begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23}\end{pmatrix}, \qquad (16)$$

where θ_{12} can be either large or small, depending on the solar neutrino oscillation solution.

Taking $w_1\simeq w_2\simeq\sqrt{1/2}$ (i.e., $m_1\simeq m_2$), one can obtain the *nearly* bimaximal mixing, which corresponds to

$$\theta_{12}\simeq\theta_{23}\simeq\pi/4 \ \ (\text{i.e.,}\ \ c_{12}=s_{12}=c_{23}=s_{23}=1/\sqrt{2}),$$

$$\sin\theta_{13}\simeq\sqrt{m_e/2m_\mu}\ \ \text{and}\ \ \delta_{13}\simeq\delta^l.$$

Then, the mixing matrix can be written as follows:

$$V^l_{\rm CKM}\simeq\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}}e^{-i\delta^l} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}\end{pmatrix}. \qquad (17)$$

Note that our mixing matrix contains possible CP-violating phase with nonzero but small

$$|V^l_{13}|(=\sin\theta_{13})\simeq\frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}}\simeq 0.05,$$

which is still consistent with the bound obtained from present CHOOZ experiment [11]. However, if it turns out that the neutrino mixing pattern is *exact* bimaximal mixing as suggested in Refs. [12,13], the element $|V^l_{13}|$ would become exactly zero and then we could not see any CP violation effects in the leptonic sector.

In the limit of small mass ratio m_1/m_2, which corresponds to the small mixing angle solution of the solar neutrino oscillation, the lepton mixing matrix (16) becomes

$$V^l_{\rm CKM}\simeq\begin{pmatrix} c_{12} & s_{12} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}}e^{-i\delta^l} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}, \qquad (18)$$

where

$$\tan\theta_{12}\simeq\sqrt{m_1/m_2+m_e/2m_\mu-\sqrt{(m_1m_e)/(2m_2m_\mu)}\cos\delta^l}.$$

We note that the mixing angle θ_{12} is correlated with the phase δ^l in this case.

III. NEUTRINO MASSES AND LEPTONIC *CP* VIOLATION

In order to determine the magnitudes of the elements of the neutrino mass matrix and the possible range of CP violation, we have to obtain numerical values of the neutrino mass eigenvalues. Although we cannot obtain those values exactly, some possible ranges of the neutrino mass eigenvalues can be estimated from the recent experimental results by making reasonable assumptions.

Most data on neutrino mixings are presented in the two-neutrino scheme. The results are expressed in $(\Delta m^2,\sin^2 2\theta)$ plot. With a proper approximation we can use the data on solar and atmospheric neutrino oscillations to make analyses for the three-neutrino scheme [14]. First we assume that the solar neutrino problems are solved by two-neutrino vacuum oscillations of $\nu_e\leftrightarrow\nu_\mu$. The survival probability for solar electron-neutrino in two-neutrino mixing scheme is given by

$$P(\nu_e\to\nu_e)=1-\sin^2 2\theta_{sol}\sin^2\left(\frac{\Delta m^2_{sol}}{4}\frac{L}{E}\right). \qquad (19)$$

In the case of $m_3^2L/E\gg 1$ and $|V^l_{13}|\ll 1$, the survival probability for electron-neutrino in the three-neutrino mixing scheme may be written as

$$P(\nu_e\to\nu_e)\simeq 1-4|V^l_{11}V^l_{12}|^2\sin^2\left(\frac{\Delta m^2_{21}}{4}\frac{L}{E}\right). \qquad (20)$$

Therefore, the mass squared difference and mixing angle for solar neutrino analysis in the two-neutrino scheme are related to the mass squared difference and the standard mixing angle, θ_{12}, in the three-neutrino scheme:

$$\Delta m^2_{sol}\simeq\Delta m^2_{21},\qquad \theta_{sol}\simeq\theta_{12}. \qquad (21)$$

If we consider the matter effect in the Sun, the survival probability for electron neutrinos, Eqs. (19) and (20), is no longer valid. However, we can still make the connections between two- and three-neutrino oscillation parameters by Eq. (21) in this situation; the mixing angle θ_{13} is small, i.e., $|V^l_{13}|\ll 1$, and the third neutrino mass m_3 is so large that just one resonance conversion between m_1 and m_2 neutrino mass states can take place. In this case the three-neutrino mixing scheme may effectively be reduced to the two-neutrino mixing scheme and Eq. (21) remains valid. If three neutrino masses are degenerate such that the second resonance conversion could not be negligible, or the mixing element $|V^l_{13}|$ is large, then we have to analyze neutrino mixing data within full three-neutrino scheme.

Likewise, we can consider the atmospheric neutrino case. The atmospheric neutrino deficit seems to be explained by oscillation between ν_μ and ν_τ with large mixing. The survival probability for atmospheric muon-neutrino in the two-neutrino mixing scheme is given by

$$P(\nu_\mu\to\nu_\mu)=1-\sin^2 2\theta_{atm}\sin^2\left(\frac{\Delta m^2_{atm}}{4}\frac{L}{E}\right). \quad (22)$$

In the case of $(m_2^2-m_1^2)L/E\ll 1$, we can write the survival probability for muon-neutrino in the three-neutrino mixing scheme as follows:

$$P(\nu_\mu\to\nu_\mu)\simeq 1-4|V^l_{23}|^2(1-|V^l_{23}|^2)\sin^2\left(\frac{\Delta m^2_{31}}{4}\frac{L}{E}\right). \quad (23)$$

The mass squared difference and mixing angle for atmospheric neutrino analysis in the two-neutrino scheme are related to the mass squared difference and the standard mixing angle, θ_{23}, in the three-neutrino scheme:

$$\Delta m^2_{atm}\simeq\Delta m^2_{31}, \quad \theta_{atm}\simeq\theta_{23}. \quad (24)$$

Recent Super-Kamiokande experiments [5] show evidence for oscillation of atmospheric neutrinos. The data exhibit a zenith angle dependent deficit of muon neutrinos, which is consistent with predictions based on the two-flavor $\nu_\mu\leftrightarrow\nu_\tau$ oscillations. At 90% confidence level the mass squared difference and mixing angle are

$$5\times10^{-4}<\Delta m^2_{atm}<6\times10^{-3} \ \ \mathrm{eV}^2, \quad (25)$$

$$0.82<\sin^2 2\theta_{atm}\leqslant 1. \quad (26)$$

The best fit values are $\Delta m^2_{atm}\simeq 2.2\times10^{-3}$ eV^2 and $\sin^2 2\theta_{atm}=1$. From the mixing matrix (5) the mixing angle θ_{23} is expressed in terms of the neutrino masses and the parameter ϵ^l as

$$\tan\theta_{23}\simeq\sqrt{\frac{\epsilon^l}{m_3-\epsilon^l}}. \quad (27)$$

We can constrain the ratio, ϵ^l/m_3, from Eqs. (24) and (26)

$$0.28\leqslant\frac{\epsilon^l}{m_3}\leqslant 0.5. \quad (28)$$

If we take the best fit value for the mass squared difference and assume the mass hierarchy of $m_1,m_2\ll m_3$, we conclude that

$$m_3\simeq\sqrt{\Delta m^2_{atm}}\simeq 4.7\times10^{-2} \ \ \mathrm{eV}, \quad (29)$$

$$1.3\times10^{-2}\leqslant\epsilon^l\leqslant 2.4\times10^{-2} \ \ \mathrm{eV}. \quad (30)$$

As is well known, there are three oscillation solutions of the solar neutrino problems in the two-neutrino scheme [4].

Large mixing angle (LMA) MSW oscillations:

$$5\times10^{-6}\leqslant\Delta m^2_{sol}\leqslant 4\times10^{-5} \ \ \mathrm{eV}^2, \quad (31)$$

$$0.4\leqslant\sin^2 2\theta_{sol}\leqslant 0.9. \quad (32)$$

Vacuum oscillations (VO):

$$5\times10^{-11}\leqslant\Delta m^2_{sol}\leqslant 10^{-10} \ \ \mathrm{eV}^2, \quad (33)$$

$$0.67\leqslant\sin^2 2\theta_{sol}\leqslant 1. \quad (34)$$

Small mixing angle (SMA) MSW oscillations:

$$3.8\times10^{-6}\leqslant\Delta m^2_{sol}\leqslant 10^{-5} \ \ \mathrm{eV}^2, \quad (35)$$

$$3.5\times10^{-3}\leqslant\sin^2 2\theta_{sol}\leqslant 1.4\times10^{-2}. \quad (36)$$

The intervals are 95% confidence level. We will not consider the MSW solution with low mass at $\Delta m^2_{sol}=7.9\times10^{-8}$ eV^2, $\sin^2 2\theta_{sol}=0.96$. All the solutions are consistent with the predictions of the standard solar model [4] and with the observed average event rates in the Chlorine (Homestake) experiments [15], Kamiokande [16], Super-Kamiokande [17], Gallium (GALLEX [18] and SAGE [19]) experiments. With these results from the solar neutrino oscillation solutions, we can investigate the bounds on the neutrino masses, m_1,m_2, and CP violation quantity, J^l_{CP}, based on the mass matrix ansatz (3). Although it is difficult to severely constrain the CP-violating phase from the results of solar and atmospheric experiments, we can get the possible ranges of magnitude of J^l_{CP} for a given nonzero CP phase. To show this in detail, we will treat three cases of the solar neutrino oscillation solutions separately.

LMA solution.

Recent experimental results from Super-Kamiokande seem to provide some encouragement for considering the LMA solution of the MSW effect [20]. The best fit values are at $\Delta m^2_{sol}\simeq 10^{-5}$eV^2 and $\sin^2 2\theta_{sol}\simeq 0.6$. The lepton flavor mixing matrix for this solution in the leading approximation is given by Eq. (17). From Eq. (13), the neutrino mixing angle θ_{12} is expressed in terms of the lepton masses, m_e,m_μ,m_1,m_2, and the CP phase δ^l. Since the LMA solution is the case of $m_1\simeq m_2$, one may ignore relatively small terms which contain the ratio m_e/m_μ in Eq. (13). Then, we approximately get

$$\theta_{sol}\simeq\theta_{12}\simeq\arctan\left(\frac{m_1}{m_2}\right), \quad (37)$$

which is in turn bounded by Eq. (32). Using the bounds of the mass squared difference (31) and mixing angle θ_{12}, we obtain numerically allowed neutrino mass bounds:

$$3.0\times10^{-4}\leqslant m_1\leqslant 2.0\times10^{-3} \ \ \mathrm{eV}, \quad (38)$$

$$2.7\times10^{-3}\leqslant m_2\leqslant 1.5\times10^{-2} \ \ \mathrm{eV}. \quad (39)$$

With the 90% confidence limit data on neutrino oscillation parameters we can estimate the possible ranges of the magnitude of the complex mixing matrix elements from the numerical analysis based on the exact form of the mixing matrix:

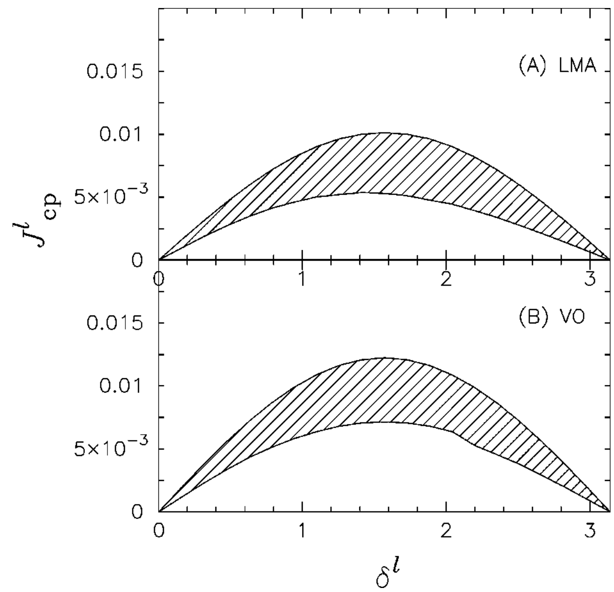

FIG. 1. The leptonic CP-violating quantity, J^l_{CP}, as a function of the CP phase, δ^l. The hatched regions are allowed by 95% C.L. mass squared difference and mixing angle from the solar and atmospheric neutrino data. We consider the possible solutions for the solar neutrino problem: (A) large mixing angle (LMA) MSW and (B) vacuum oscillation (VO).

$$|V_{\rm LMA}|=\begin{pmatrix} 0.82\text{–}0.94 & 0.35\text{–}0.55 & 0.01\text{–}0.10 \\ 0.25\text{–}0.50 & 0.55\text{–}0.77 & 0.56\text{–}0.70 \\ 0.14\text{–}0.34 & 0.50\text{–}0.65 & 0.70\text{–}0.82 \end{pmatrix}. \quad (40)$$

From these results, we can calculate the quantity J^l_{CP} as a function of the CP phase δ^l. In Fig. 1(a), we present our prediction for the allowed range of J^l_{CP} as a function of δ^l, which is consistent with the solar and atmospheric neutrino experimental results.

VO solution.

This solution shows that there are well-separated two mass squared difference scales, $\Delta m^2_{atm}\simeq 2.2\times 10^{-3}$ eV^2 and $\Delta m^2_{sol}\simeq 8\times 10^{-11}$ eV^2. The mixing angle θ_{12} is also determined by the neutrino masses m_1 and m_2 as in the case of LMA solution (37). From the constraints Eqs. (33) and (34), we get, at best, the lower bounds on neutrino masses as follows:

$$m_1 \geqslant 0.24\times 10^{-5}\ \text{eV}, \quad m_2 \geqslant 0.93\times 10^{-5}\ \text{eV}. \quad (41)$$

The limits on magnitudes of the elements of the complex mixing matrix for this solution are

$$|V_{\rm VO}|=\begin{pmatrix} 0.70\text{–}0.90 & 0.45\text{–}0.71 & 0.02\text{–}0.06 \\ 0.35\text{–}0.60 & 0.47\text{–}0.75 & 0.54\text{–}0.72 \\ 0.20\text{–}0.50 & 0.40\text{–}0.65 & 0.70\text{–}0.84 \end{pmatrix}. \quad (42)$$

Since the rephasing-invariant J^l_{CP} given in Eqs. (11) actually depends on the ratio m_1/m_2 and the parameter ϵ^l, we can calculate the numerical values of J^l_{CP} with the help of Eqs. (28), (34), and (37). The results are shown in Fig. 1(b). As

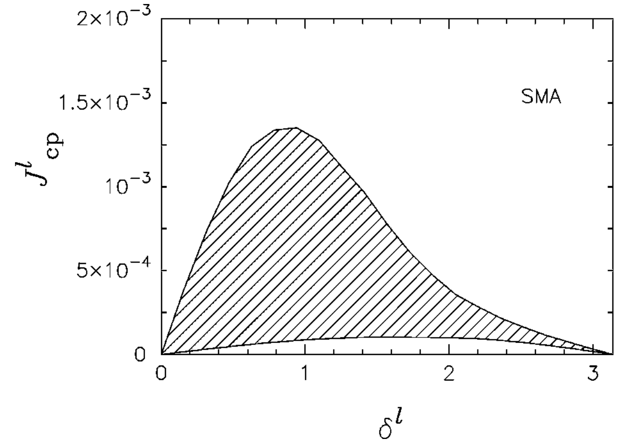

FIG. 2. The leptonic CP-violating quantity, J^l_{CP}, as a function of the CP phase, δ^l. The hatched region is allowed by 95% C.L. mass squared difference and mixing angle from the solar and atmospheric neutrino data. We consider one of the possible solutions for the solar neutrino problem: small mixing angle (SMA) MSW.

one can see from Fig. 1(b), the allowed range of J^l_{CP} for the VO solution is almost the same as that for the LMA solution. The reason is that both solutions have almost the same neutrino mixing angle θ_{12} and our ansatz leads to those solutions when we take the mass hierarchy $m_1\simeq m_2$. Therefore the mixing matrix may also be accommodated by Eq. (17) in the leading approximation as in the case of LMA solution.

SMA solution.

The small mixing angle θ_{12} implies small mass ratio, m_1/m_2, in Eq. (5). Different from the above two cases, the lepton mixing matrix for the SMA solution in the leading approximation is given by Eq. (18). In this case the angle depends sensitively on the phase δ^l as well as on the ratio m_1/m_2. Note that the expression (37) does not hold in the extreme mass hierarchical case of $m_2\gg m_1$, because we cannot neglect the m_e/m_μ terms in Eq. (13). Also, the dependence of CP phase should be taken into account when we calculate the mixing angle $\theta_{12}\simeq\theta_{sol}$. From the numerical analysis based on the exact form of the mixing matrix, it has been shown that the mass m_1 may have small value without a lower bound in the allowed parameter space given by Eqs. (35) and (36). The upper bound of mass m_1 depends on the value of CP-violating phase δ^l. When δ^l is in the range of $0<\delta^l<\pi/2$, the upper bound of m_1 may be up to 4.2×10^{-5} eV. As δ^l approaches π, the upper bound of m_1 becomes smaller, around 3.0×10^{-7} eV. From Eq. (35), the lower mass bound of m_2 is to be around 1.9×10^{-3} eV. The limits on magnitudes of the elements of the complex mixing matrix for this solution are

$$|V_{\rm SMA}|=\begin{pmatrix} 0.98\text{–}0.99 & 0.03\text{–}0.06 & 0.03\text{–}0.05 \\ 0.03\text{–}0.08 & 0.70\text{–}0.84 & 0.54\text{–}0.71 \\ 0.01\text{–}0.05 & 0.54\text{–}0.71 & 0.70\text{–}0.83 \end{pmatrix}. \quad (43)$$

Figure 2 shows the rephasing-invariant quantity J^l_{CP} for the SMA solution as function of the CP phase δ^l. The allowed range of J^l_{CP} is presented by the hatched region which comes from the constraints Eqs. (35) and (36). The shape of allowed region is different from that of large mixing of solar neutrinos. The maximum value of J^l_{CP} for the SMA solution is 1.3×10^{-3}, while that for LMA and VO is of order ~ 0.01. In particular, as can be seen from Fig. 2, the magnitude of J^l_{CP} is suppressed and severely constrained for the range of $\pi/2<\delta^l<\pi$. Also, somewhat broad range of J^l_{CP} for $\delta^l\sim 1$ is obtained.

To summarize, we analyzed neutrino masses and mixings as the solutions of the solar neutrino problems and atmospheric neutrino deficits based on a mass matrix ansatz. Recent Super-Kamiokande results for atmospheric neutrino showed that the muon neutrino deficits may be explained by large mixing between $\nu_\mu \leftrightarrow \nu_\tau$. The solar neutrino problems have three possible solutions: small mixing MSW, large mixing MSW, and just-so vacuum oscillation solutions. Depending on the solutions to the solar neutrino problems, we have three possible mixing matrices in the three-neutrino scheme. For each case we investigated the neutrino mass bounds, the magnitudes of mixing matrix elements, and possible nonvanishing CP-violating rephasing-invariant quantity J^l_{CP}. We conclude that LMA-MSW and VO solutions may come from the mass matrix ansatz with similar mass hierarchy: $m_1 \simeq m_2 \ll m_3$. And J^l_{CP} also has almost the same magnitude in the two cases, and may reach values up to 0.012. The origin of the SMA-MSW solution may be attributed to the mass hierarchy: $m_1 \ll m_2 \ll m_3$ with our mass matrix ansatz. The magnitude of J^l_{CP} depends on the CP phase δ^l. In the range of $0\leqslant\delta^l\leqslant\pi/2$, the value of J^l_{CP} may be up to 1.3×10^{-3}, which is small compared to the LMA-MSW or VO solution. In $\pi/2<\delta^l<\pi$, J^l_{CP} is even more suppressed.

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